Lesson 1–2

Topic: Number and place value

Exploring square numbers

Lesson concepts

Number — Square and triangular

Lesson notes

Students will:

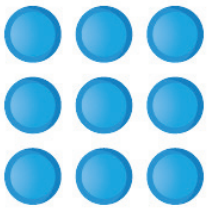
- create square number arrays
- investigate square numbers.

It is not important for students to know whether rows or columns are written first when describing arrays. It is, however, important for students to know that a row is the horizontal part of an array and that a column is the vertical part of an array.

Lesson answers

1. Answers will vary depending on the student's array; however, sample answers have been provided for the example array 2 × 4:
   a. Rectangle
   b. 2
   c. 4
   d. 4 × 2, 8 × 1, 1 × 8

2. Answers will vary depending on the student's array; however, sample answers have been provided for the example array 3 × 3:
   a. Square
   b. 3
   c. 3
   d. 1 × 9 , 9 × 1



3.

Number	Eight	Nine	Four	Twelve	Sixteen
Shape	 Rectangle	 Square			
Factors	1 × 8 8 × 1 2 × 4 4 × 2	1 × 9 9 × 1 3 × 3	2 × 2 1 × 4 4 × 1	3 × 4 4 × 3 1 × 12 12 × 1 2 × 6 6 × 2	4 × 4 1 × 16 16 × 1 8 × 2 2 × 8

4. a. The next square number is 7 × 7. 7 × 7 equals 49.

   b. For example: The arrays grow by one counter in each row and column each time.

5.

Number	Number multiplied by itself	Square number
1	1 × 1 = 1	1
2	2 × 2 = 4	4
3	3 × 3 = 9	9
4	4 × 4 = 16	16
5	5 × 5 = 25	25
6	6 × 6 = 36	36
7	7 × 7 = 49	49
8	8 × 8 = 64	64
9	9 × 9 = 81	81
10	10 × 10 = 100	100

6. a. 1, 4, 9, 16, 25, 36, 49.

   b. For example: I can see that each square I cut out is growing by one row/column on each side.

7.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

8. For example: Yes, I found the same numbers as in question five.

Lesson 3

Topic: Number and place value

Exploring square numbers

Lesson concepts

Number — Square and triangular

Lesson notes

Students will:

- represent triangular numbers
- investigate triangular numbers.

Lesson answers

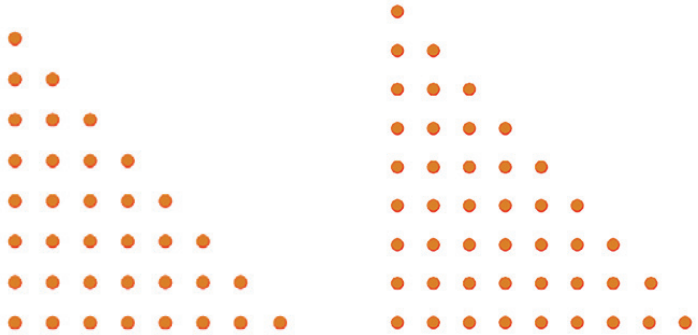
1 a. 

b. 

c. The number six.

d. Three counters.

2. a. Each row had an extra one. 15 had five rows, 21 had six rows and 28 had seven rows.
   b. 15, 21, and 28
3. a. For example: Each row is growing by an extra one counter in the final column.
4. Students watch the **Video — Triangular numbers.**
   a. For example: 36, 45
   b. For example:
   36 and 45



   c. Yes, my predictions were correct. The next two numbers were 36 and 45.
   d. 36 had a bottom row of eight, one more than the previous number 28, which had a bottom row of seven. The next number, 45, had a bottom row of nine, which followed eight (36).

5.

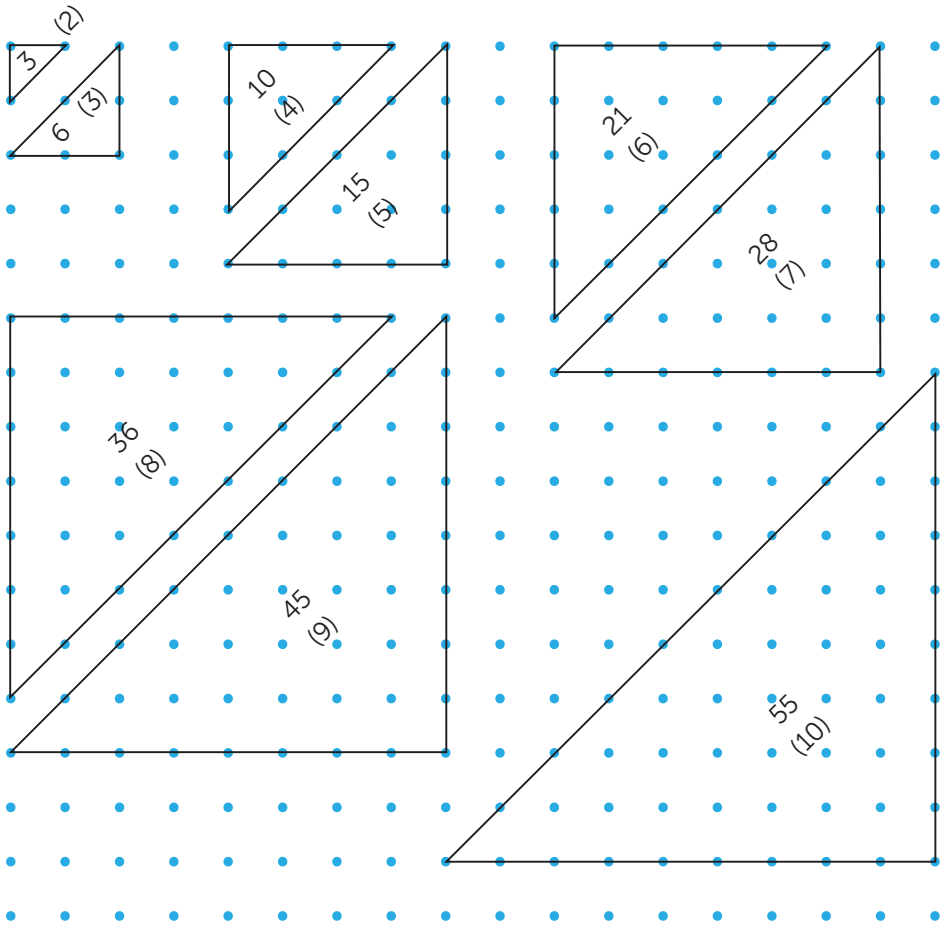
Term	Triangular number	Number of extra counters needed for next triangular number
1	1	+2
2	3	+3
3	6	+4
4	10	+5
5	15	+6
6	21	+7
7	28	+8
8	36	+9
9	45	+10
10	55	+11
11	66	+12
12	78	+13
13	91	+14
14	105	+15
15	120	+16

6. a. Activity Sheet – Hundred board

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100					105				

6. b. For Example: Triangular numbers

Centimetre dot paper



c. For example: Each triangle has an extra row and each new row grows by one.

Lesson 4

Topic: Number and place value

The handshake problem

There are many solutions on the nrich site recommended. The solution is the number of people times one less than the number of people then divided by 2. We divide by 2 as me shaking your hand is the same as you shaking mine.

Lesson 5

Topic: Number and place value

Exploring the relationship between square and triangular numbers

Lesson concepts

Number — Square and triangular

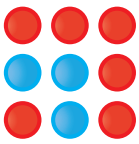
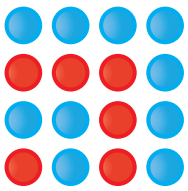
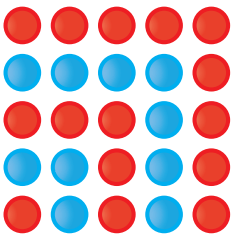
Lesson notes

Students will:

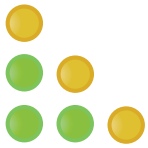
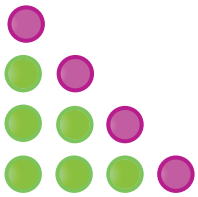
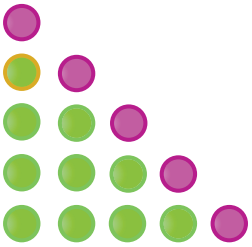
- review square and triangular numbers
- investigate the relationship between square and triangular numbers.

Lesson answers

1. a. A square number is the product of a number multiplied by itself. A square number can form a square-shaped array.
   b. If it is square, the number will be able to form a square array or be a product of a number multiplied by itself.
   c. 1, 4, 9, 16, 25
   d.

First	Second	Third	Fourth	Fifth
				

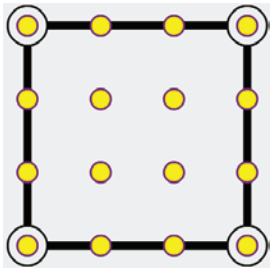
2. a. A triangular number is a number that creates a triangle-shaped array.
   b. A triangular number creates a triangular shape. Each row should grow by one.
   c. 1, 3, 6, 10, 15
   d.

First	Second	Third	Fourth	Fifth
				

3. a. There are 11 square numbers between 1 and 100. There are 13 triangular numbers between 1 and 100. Therefore there are two more triangular numbers.
   b. For example: I think there are more triangular numbers because the pattern grows by an additional row; however, square numbers grow by multiplying numbers by themselves, therefore growing larger quicker.

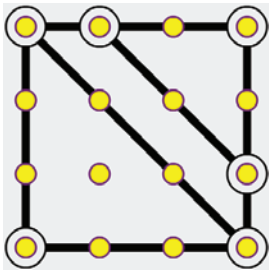
4. Students open the **Learning object — Geoboard.**

   a.



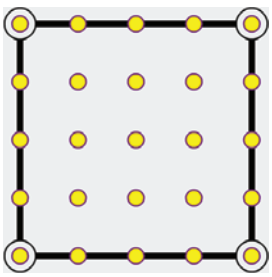
   b. For example:
   Yes, I can find a triangle or a parallelogram.

   c. For example:

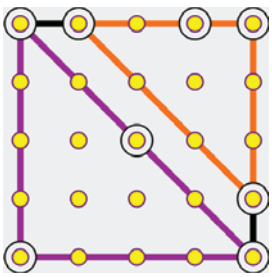


   d. Two small triangles fit into a square.
   One triangle is smaller than the other.

5 a.



   b.



   c. Yes, 10 and 15

   d. 10 + 15 = 25

6. a.

Smaller triangular number	Square number	Square number	Verifies the statement? Y or N
1	3	4	Y
3	6	9	Y
6	10	16	Y
10	15	25	Y
15	21	36	Y
21	28	49	Y
28	36	64	Y
36	45	81	Y
45	55	100	Y

6. b. For example: Completing this activity proved that two consecutive triangles do in fact add to create a square number.

Lesson 6

Topic: Number and place value

Investigating numbers less than zero

Lesson concepts

Real numbers — Integers

Lesson notes

Students will:

- position positive and negative whole numbers on a number line.

Lesson answers

1. For example:
   a. Below sea level means below the top of where the ocean would be if it was able to cover the planet without any barriers. These barriers are constructions such as dykes in the Netherlands, dams, dunes, mountain ranges, walls and many others.
   b. Elevators go high up into the part of the building above the ground, as well as the basement, which is below the ground.
2. Students watch the Video — Integers on a number line.
3. a. Students construct a number line, one metre long, on the ground.
   b. Students place the number 0 on the number line.
   c. Students place the numbers 1 to 10 on the number line.

4. a. 7
   b. 3
   c. 234
   d. You would be walking uo the number line.
   e. The numbers increase in value.
   f. The numbers decrease in value.

5. Students reposition numbers on the number line.

6. a. Yes, for example, because there has to be numbers to show less than zero.
   b. –1
   c. 1
   d. 1
   e. 2
   f. 1 is larger because it is a positive number and it is more than zero.

7. Answers:
   a. –4. It is less than 4.
   b. –3. It is less than 3.
   c. Students place remaining numbers on number line.
   d. 5
   e. 3
   f. 8
   g. 7. This means 1 is 7 greater than –6.
   h. 4
   i. 8 is greater than –9 because 8 is a positive number.

Reflection

For example: Yes, when I owed my brother $5.

Lesson 7

Topic: Number and place value

Solving problems on a number line

Lesson concepts

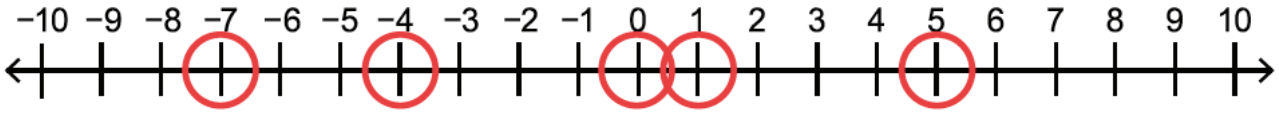
Real numbers — Integers

Lesson notes

Students will:

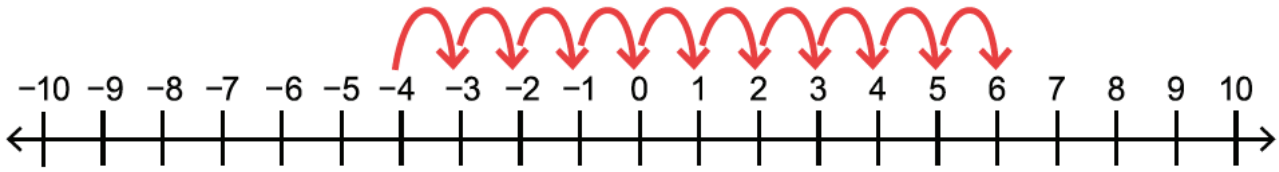
- use a number line to solve problems involving positive and negative numbers.

Lesson answers

1. Answers will vary depending on the five cards drawn. Check that they are positioned correctly on the number line with the largest negative number furthest to the left and the largest positive number furthest to the right.
   Sample response:
   a. No answer required.
   b. For example: −4, 5, 0, −7, 1
   c. 

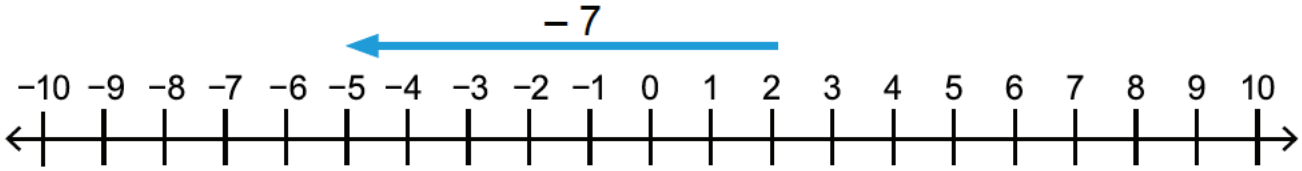
   d. For example: −7, −4, 0, 1, 5

2. Students watch the **Video — Use a number line to solve problems.**

3. **Problem 1**
   a. 

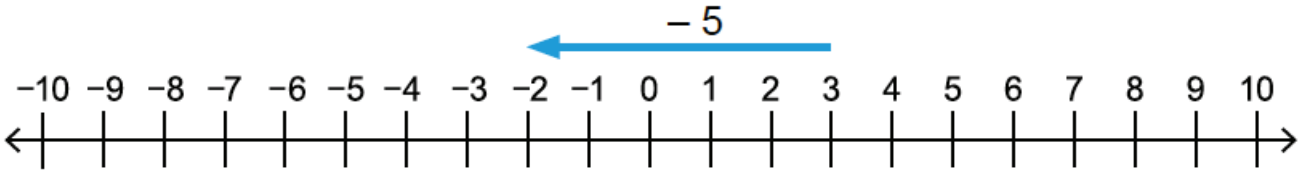
   b. $6
   c. If the numbers are getting bigger you have to move right.
   d. More.
   For example: He started with a negative amount and now he has a positive amount.
   e. −4 + 10 = 6

4. **Problem 2**



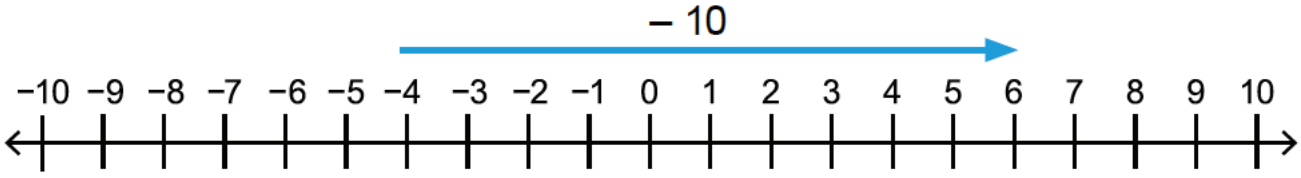
a. The temperature is now −5 °C
b. 7
c. Left
d. Yes
e. 2 – 7 = −5

5. **Problem 3**



a. −2 metres
b. Left
c. 3 – 5 = −2

6. **Problem 4**



a. Floor 6
b. −4
c. Right
d. −4 + 10 = 6

Lesson 8

Topic: Number and place value

Locating positive and negative numbers on a line

Lesson concepts

Real numbers — Integers

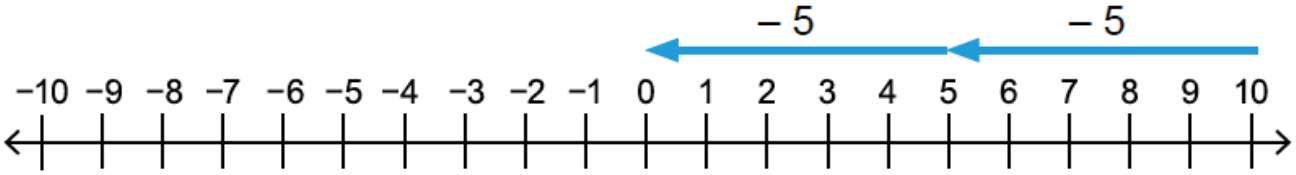
Lesson notes

Students will:

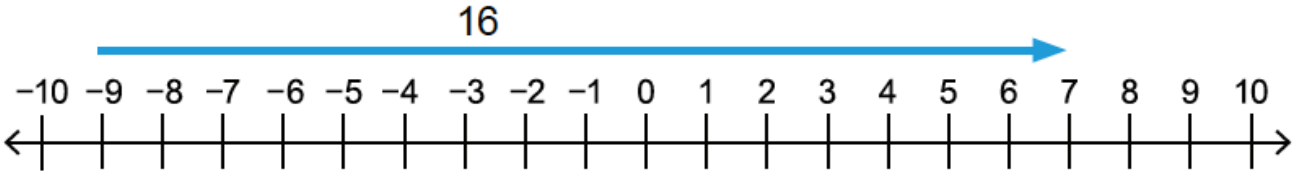
- demonstrate understanding of positive and negative numbers and their position on a number line.

Lesson answers

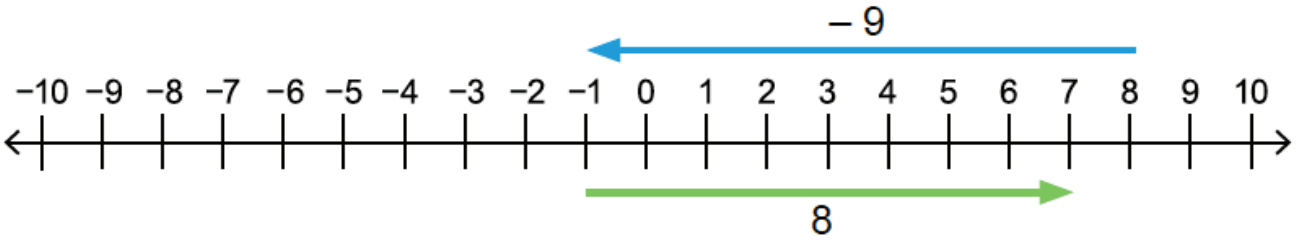
1. Students watch the **Video — Integers on a number line.**
2. b. 10 – 5 – 5 = 0



c. –9 + 16 = 7

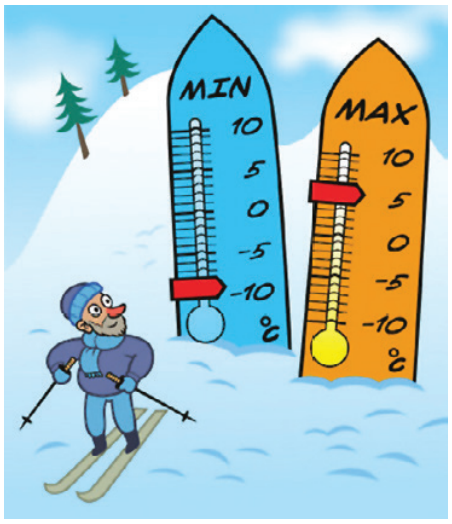


d. 8 – 9 + 8 = 7



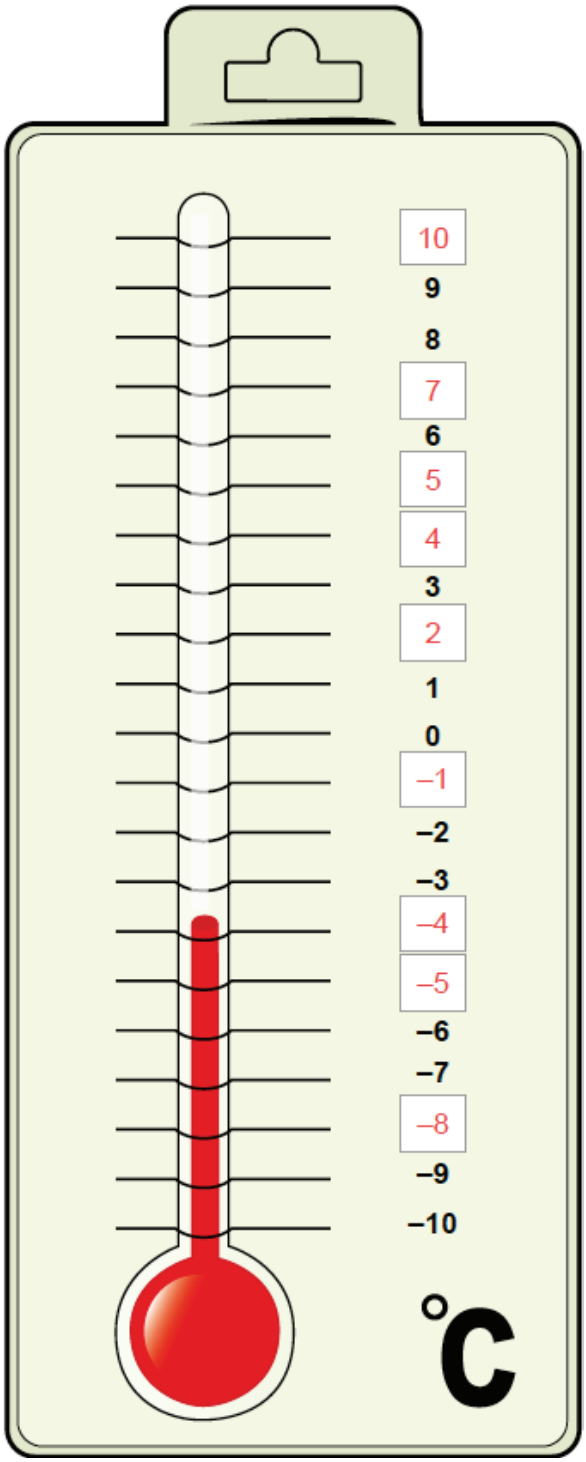
3. a. –5 and 5
   b. 8
   c. Left
   d. Bigger
   e. 5
4. **Sheet 5 — Thredbo thermometers** (attached).

Thredbo thermometers



Thredbo is in the New South Wales Snowy Mountains at the foot of Mount Kosciuszko. It is famous for its skiing in winter. Each day, visitors can read the maximum and minimum temperatures on giant thermometers. Your task is to answer the questions below about thermometers and temperatures.

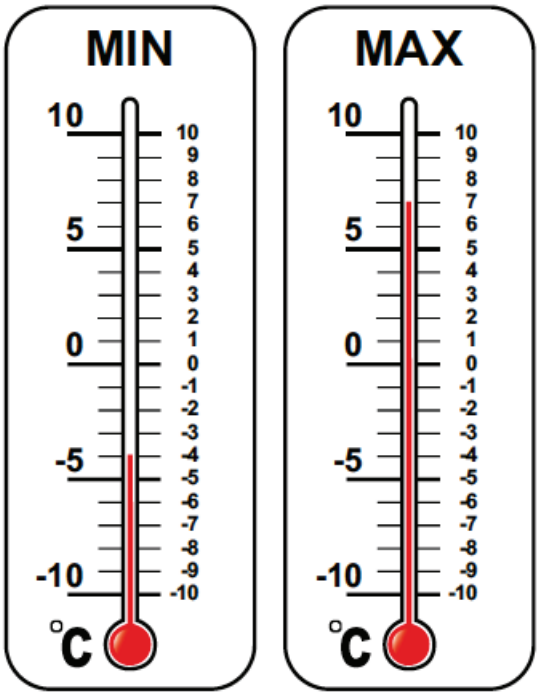
1. Look at the numbers on the right-hand side of the thermometer.
   Fill in the missing numbers on or beside the thermometer.



2. Colour the thermometers to show the minimum and maximum temperatures for Monday and Tuesday.
Fill in the missing numbers on or beside the thermometer.

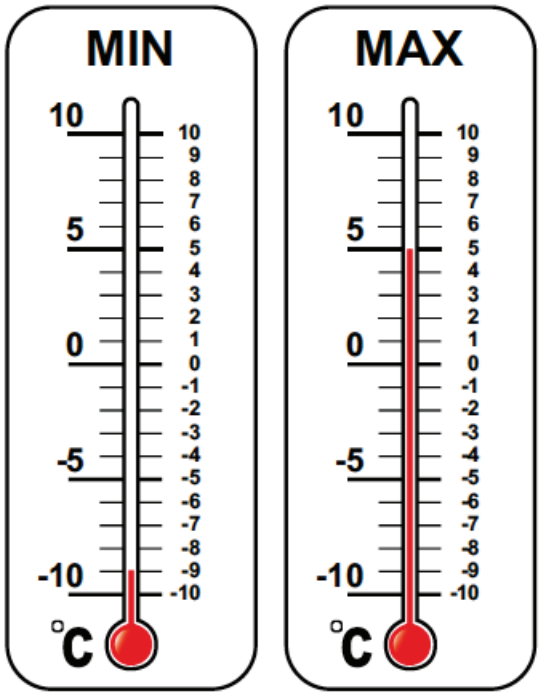
Monday

Minimum: – 4° Maximum: 7°



Tuesday

Minimum: – 9° Maximum: 5°



3. The minimum temperature on Wednesday is slightly warmer than on Tuesday. What is the minimum temperature on Wednesday? (Circle one).

–10°C 0°C (– 8°C) –1°C 8°C

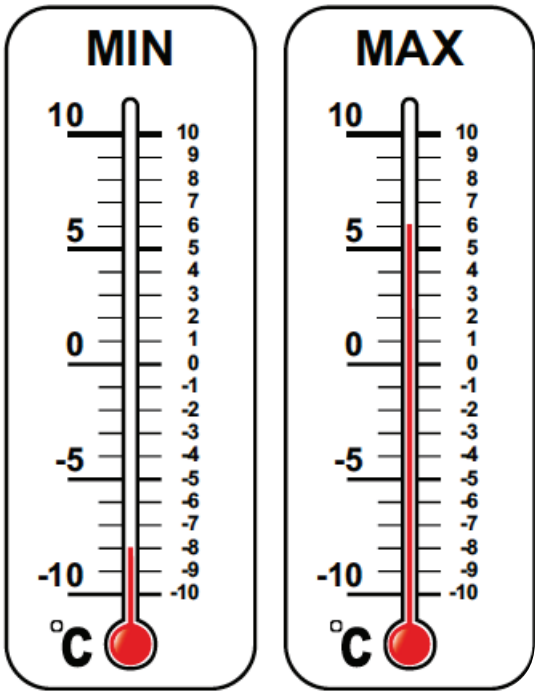
Likewise, the maximum temperature on Wednesday is slightly warmer than on Tuesday.

What is the maximum temperature on Wednesday? (Circle one).

–10°C 10°C – 8°C –1°C (6°C)

Show Wednesday's minimum and maximum temperatures on the thermometers.

Wednesday's temperatures

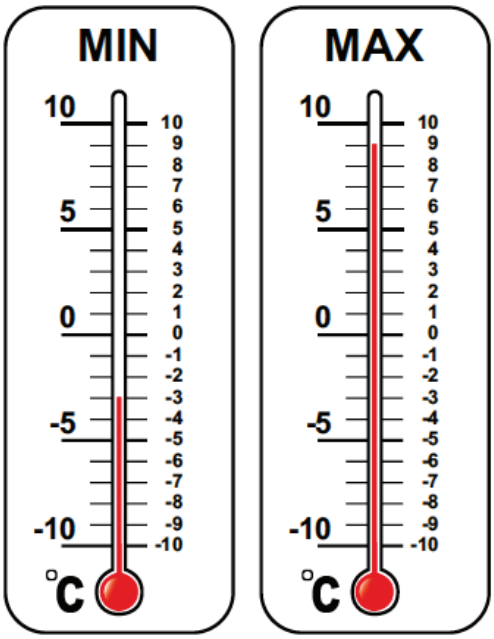


4. On the following thermometers show the maximum temperatures and the minimum temperatures for Thursday and Friday.

Thursday

Minimum: 2 degrees warmer than – 5 °C

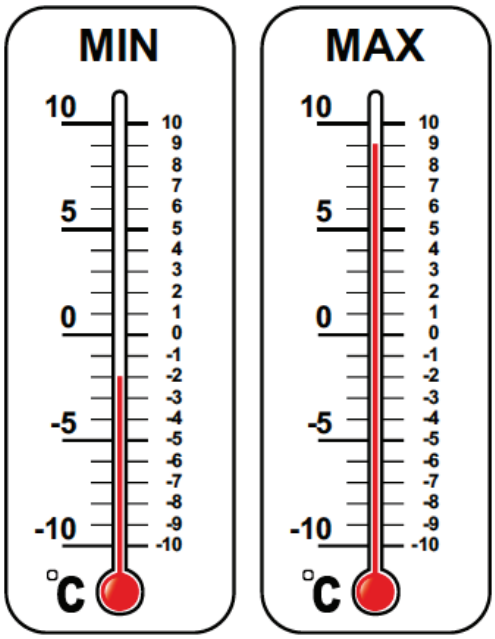
Maximum: 1 degree cooler than 10 degrees



Friday

Minimum: 2 degrees colder than 0 °C

Maximum: 1 degree warmer than 8 degrees

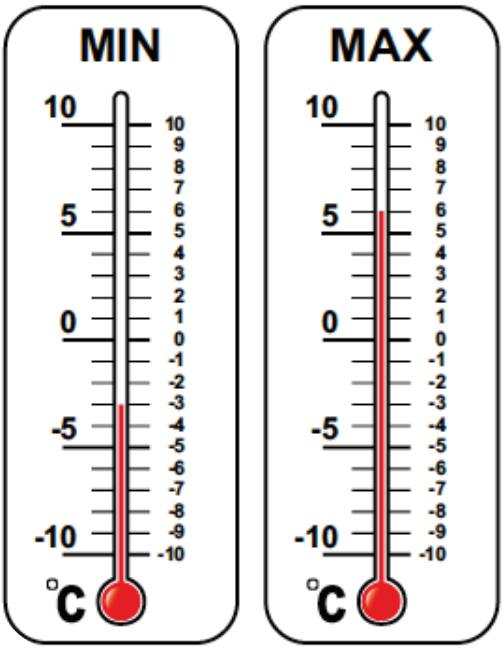


5. Here are the thermometers for Saturday and Sunday. What is the minimum and maximum temperature for each day?

Thursday

Minimum: – 3°C degrees

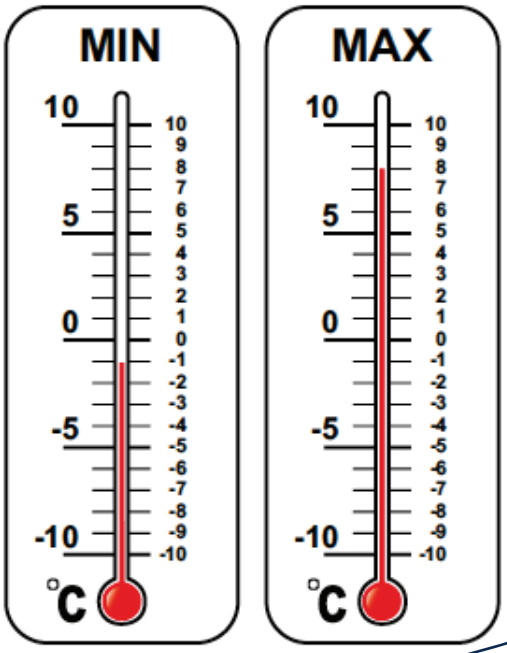
Maximum: 6°C degrees



Friday

Minimum: – 1°C degrees

Maximum: 8°C degrees



6. Explain how and when you might use negative numbers in real life. Personal response required.
   For example:
   - when I owe money to my brother
   - when the temperature gets below zero at night in winter
   - bank accounts.