Lesson 1

Topic: Patterns and algebra

Consolidating the distributive law

Lesson concepts

- **Equivalence** — Distributive law
- **Representations** — Algebraic conventions

Lesson notes

Today students will:

- review terminology — expressions, terms, pronumerals
- explore scenarios using principles of the distributive law
- expand expressions of the form $a(b + c)$.

Lesson answers

1.

Expression	Terms	Pronumeral(s)
$7f - 11s$	$7f$, $11s$	f, s
$8g + 2d - 3$	$8g$, $2d$, -3	g, d
$9x^2 + 6x + 15$	$9x^2$, $6x$, 15	x
$8\ (2g + 7)$	8, $2g$, 7	g

2. Students watch the **Video — Modelling the distributive law: Preparing for a party — Part 1.**
3. Methods A, C and E from the video could be used to correctly calculate the total lollies.
4. Students watch the **Video — Modelling the distributive law: Preparing for a party — Part 2.**
5. a) $32\ (5 + 6) = 32 \times 5 + 32 \times 6$
   b) $5\ (2 - 9) = 5 \times 2 - 5 \times 9$ or $5 \times 2 + 5 \times -9$
   c) $14\ (5 + 6 + 11) = 14 \times 5 + 14 \times 6 + 14 \times 11$

6 a) $144khw$

b) $72hjx$

c) $-1\,200ejklz$

7. a) $8h^2$

b) $-27h^2j$

c) $40k^3$

8. Students watch the **Video — Methods to expand $a(b + c)$.**

9. a) $4(c + 10)$

$= 4 \times c + 4 \times 10$

$= 4c + 40$

b) $5(2g - 9)$

$= 5 \times 2g - 5 \times 9$

$= 10g - 45$

c) $-9(5y - 6)$

$= -9 \times 5y - -9 \times 6$

$= -45y + 54$

d) $x(x + 8)$

$= x \times x + x \times 8$

$= x^2 + 8x$

e) $-5t(-2e + 3)$

$= -5t \times -2e + -5t \times 3$

$= 10et - 15t$

f) $-4r(2r^2 - 6)$

$= -4r \times 2r^2 - -4r \times 6$

$= -8r^3 + 24r$

Lesson 2

Topic: Patterns and algebra

Factorising expressions

Lesson concepts

- **Equivalence** — Distributive law
- **Representations** — Algebraic conventions

Lesson notes

Today students will:

- review factors and highest common factors
- factorise expressions in the form of $ab + ac$ where a is the highest common factor.

Lesson answers

1. Note: HCF = Highest common factor.
   a) 20, 35
   Factors of 20 = 1, 2, 4, 5, 10, and 20
   Factors of 35 = 1, 5, 7, 35 (Common factors are underlined)
   HCF is 5
   b) 60, 12, 24
   Factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
   Factors of 12 are 1, 2, 3, 4, 6, 12
   Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24 (Common factors are underlined)
   HCF is 12
   c) 26, 39, 130
   Factors of 26 are 1, 2, 13, 26
   Factors of 39 are 1, 3, 13, 39
   Factors of 130 are 1, 2, 5, 10, 13, 26, 65, 130 (Common factors are underlined)
   HCF is 13

2.

a) $80fg$, $20g$

Factors of $80fg$ are f, g, 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

Factors of $20g$ are g, 1, 2, 4, 5, 10, 20 (Common factors are underlined)

HCF is $20g$

b) 66, $12a$, $30a$

Factors of 66 are 1, 2, 3, 6, 11, 22, 33, 66

Factors of $12a$ are a, 1, 2, 3, 4, 6, 12

Factors of $30a$ are a, 1, 2, 3, 5, 6, 10, 15, 30 (Common factors are underlined)

HCF is 6

c) $18a$, $21a$, $9a$

Factors of $18a$ are a, 1, 2, 3, 6, 9, 18

Factors of $21a$ are a, 1, 3, 7, 21

Factors of $9a$ are a, 1, 3, 9 (Common factors are underlined)

HCF is $3a$

d) $24a^2$, $60ag$, $36adg$

Factors of $24a^2$ are a, 1, 2, 3, 4, 6, 8, 12, 24

Factors of $60ag$ are a, g, 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Factors of $36adg$ are a, d, g, 1, 2, 3, 4, 6, 8, 12, 18, 36 (Common factors are underlined)

HCF is $12a$

3. a) $18d + 27$

Factors of $18d$ are d, 1, 2, 3, 6, 9, 18

Factors of 27 are 1, 3, 9, 27

The HCF is 9

$18d \div 9 = 2d$

$27 \div 9 = 3$

$18d + 27 = 9(2d + 3)$

b) $6a - 9 = 3(2a - 3)$

c) $x^2 + 8x = x(x + 8)$

d) $-6x^2 + 9x = 3x(-2x + 3)$ **or** $3x(2x - 3)$

e) $8ab^2 + 12ab = 4ab(2b + 3)$

f) $-24g^2 - 32g = -8g(3g + 4)$

g) $120b^2g + 72abg + 48bg^2 = 24bg(5b + 3a + 2g)$

h) $-136g^2z - 51gy + 34gz = 17g(-8gz - 3y + 2z)$

4. a) The expression: $54df + 162fg = 27f\,(2d + 6g)$ has been factorised incorrectly as the HCF is not $27f$. The HCF is $\underline{54f}$.

   b) Therefore the expression should have been factorised to: $54df + 162fg = 54f\,(d + 3g)$.

5. a) $x(x+3)$

   b) x^2+3x

6. Expand area expression $6x\,(6x + 2z)$

   $= 36x^2 + 12xz$

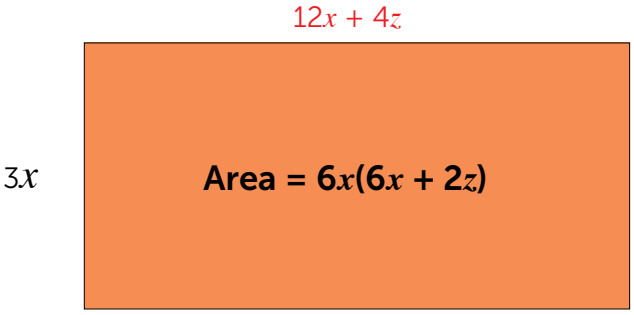
   Find length dimension Length × Width = Area

   $3x \times ? = (36x^2 + 12xz)$

   $3x \times ? = 3x\,(12x + 4z)$

   $\therefore$ Width $= 12x + 4z$

   The length of the rectangle is $12x + 4z$



Note: $4(3x + z)$ is also a valid expression for the length.

Lesson 3

Topic: Patterns and algebra

Expanding binomial expressions 1

Lesson concepts

- **Equivalence** — Distributive law
- **Equivalence** — Simplifying expressions

Lesson notes

Today students will:

- review simplifying like terms
- review the area model strategy for multiplying multi-digit pairs of numbers
- expand binomial expressions in the form of $(x + a)(x + b)$.

Lesson answers

1. a) The like terms are 8x and 4x.

   $\therefore x^2 + 8x + 4x + 32 = x^2 + 12x + 32$

   b) $5x^2 + 3x + 3$

   c) $8g + 11h - 7gh$

   d) $6xy + 4x + 5y$

2. Students watch the **Video — Using an area model to multiply numbers.**

3. a) 53 × 38

×	50	3
30	50 × 30 = 1,500	3 × 30 = 90
8	50 × 8 = 400	3 × 8 = 24

53 × 38

= 1,500 + 400 + 90 + 24

= 2,014

b) 89 × 21

×	80	9
20	80 × 20 = 1,600	9 × 20 = 180
1	80 × 1 = 80	9 × 1 = 9

89 × 21
= 1,600 + 80 + 180 + 9
= 1,869

c) 47 × 34

×	40	7
30	40 × 30 = 1,200	7 × 30 = 210
4	40 × 4 = 160	7 × 4 = 28

47 × 34
= 1,200 + 160 + 210 + 28
= 1,598

4. Students watch the **Video — Binomial expansion using an area model.**

5.

a) $(x + 4)(x + 3)$

×	x	4
x	$x \times x = x^2$	$4 \times x = 4x$
3	$3 \times x = 3x$	$4 \times 3 = 12$

$(x + 4)(x + 3)$
$= x^2 + 3x + 4x + 12$
$= x^2 + 7x + 12$

b) $(x + 3)(x + 7)$

×	x	3
x	$x \times x = x^2$	$3 \times x = 3x$
7	$7 \times x = 7x$	$3 \times 7 = 21$

$(x + 3)(x + 7)$
$= x^2 + 7x + 3x + 21$
$= x^2 + 10x + 21$

c) $(p + 9)(p + 11)$

×	p	9
9	$p \times p = p^2$	$9 \times p = 9p$
11	$11 \times p = 11p$	$9 \times 11 = 99$

$(p + 9)(p + 11)$

$= p^2 + 11p + 9p + 99$

$= p^2 + 20p + 99$

d) $(t + 4)(t - 2)$

×	t	4
t	$t \times t = t^2$	$4 \times t = 4t$
-2	$-2 \times t = -2t$	$4 \times -2 = -8$

$(t + 4)(t - 2)$

$= t^2 - 2t + 4t - 8$

$= t^2 + 2t - 8$

e) $(h + 12)(h - 7)$

×	h	12
h	$h \times h = h^2$	$12 \times h = 12h$
-7	$-7 \times h = -7h$	$12 \times -7 = -84$

$(h + 12)(h - 7)$

$= h^2 - 7h + 12h - 84$

$= h^2 + 5h - 84$

Lesson 4

Topic: Patterns and algebra

Expanding binomial expressions 2

Lesson concepts

- **Equivalence** — Distributive law
- **Equivalence** — Simplifying expressions

Lesson notes

Today students will:

- expand binomials

Lesson answers

1.

a) $(k + 4)(k + 5)$

×	k	4
k	$k \times k = k^2$	$4 \times k = 4k$
5	$5 \times k = 5k$	$4 \times 5 = 20$

$(k + 4)(k + 5)$

$= k^2 + 5k + 4k + 20$

$= k^2 + 9k + 20$

2. a) $(y + 5)(y - 3)$

×	y	5
y	$y \times y = y^2$	$5 \times y = 5y$
-3	$-3 \times y = -3y$	$-3 \times 5 = -15$

$(y + 5)(y - 3)$

$= y^2 + 5y - 3y - 15$

$= y^2 + 2y - 15$

b) $(t - 3)(t - 7)$

×	t	-3
t	$t \times t = t^2$	$3 \times t = -3t$
-7	$-7 \times t = -7t$	$-7 \times -3 = 21$

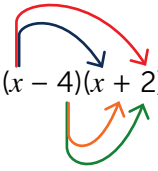
$(t - 3)(t - 7)$

$= t^2 - 7t - 3t + 21$

$= t^2 - 10t + 21$

3. Students watch the **Video — Binomial expansion using algebra.**
4. Binomial expansion using FOIL

a) $(x - 4)(x + 2)$



$(x - 4)(x + 2)$

F $x \times x = x^2$

O $x \times 2 = 2x$

I $-4 \times x = -4x$

L $-4 \times 2 = -8$

$(x - 4)(x + 2)$

$= x^2 + 2x - 4x - 8$

$= x^2 - 2x - 8$

b) $(h + 3)(h + 6)$

$= h^2 + 6h + 3h + 18$

$= h^2 + 9h + 18$

c) $(m - 2)(m - 4)$

$= m^2 - 4m - 2m + 8$

$= m^2 - 6m + 8$

d) $(2x + 3)(x + 7)$

$= 2x^2 + 14x + 3x + 21$

$= 2x^2 + 17x + 21$

e) $(3y - 4)(y + 2)$

$= 3y^2 + 6y - 4y - 8$

$= 3y^2 + 2y - 8$

f) $(r^2 + 3)(r^2 - 5)$

$= r^4 - 5r^2 + 3r^2 - 15$

$= r^4 - 2r^2 - 15$

5. **Sheet 1 — Expanding binomials** (attached).

Lesson 5

Topic: Patterns and algebra

Sketching non-linear graphs

Lesson concepts

Representations — Non-linear models

Lesson notes

Today students will:

- compare linear and non-linear relations
- sketch non-linear relations by hand.

Lesson answers

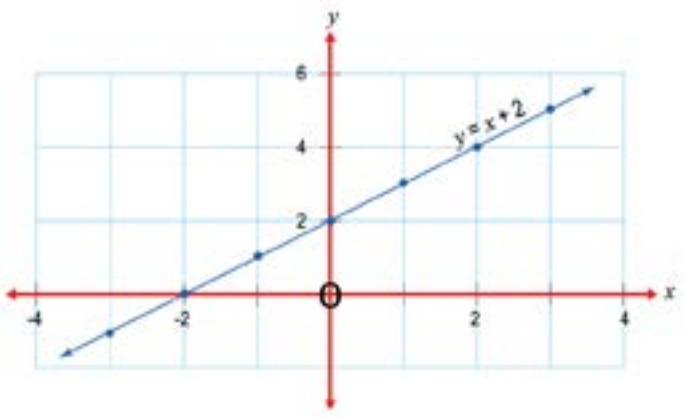
1. A number plane or Cartesian plane includes a vertical y-axis and a horizontal x-axis intersecting at the origin O.

   There are 4 quadrants.

   The point (0, 0) is called the origin.

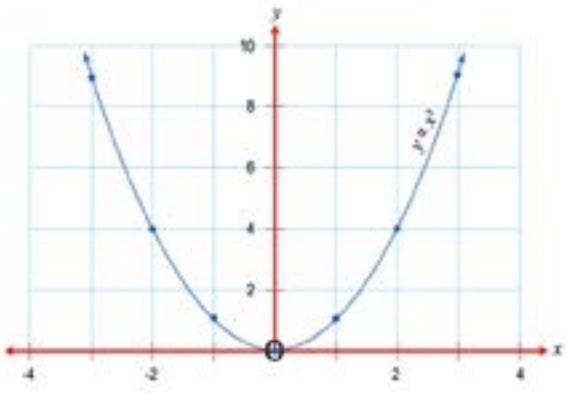
2. a) $y = x + 2$

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5



b) $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

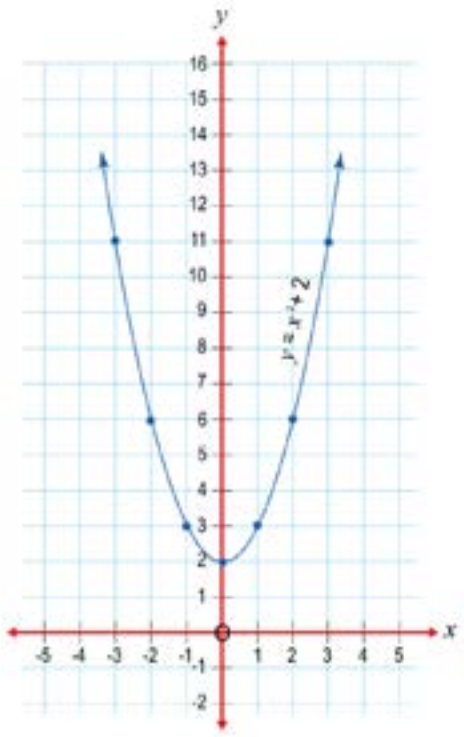


c) The first graph is linear, it is a straight line. The second graph is non-linear, it is not a straight line.

3. Students watch the **Video — How to plot a parabolic function.**

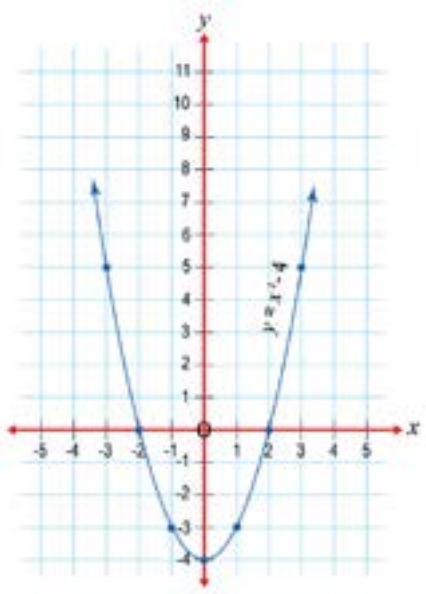
4-5. Answers for **Sheet 3 — Sketching parabolic functions:**

Question 1 $y = x^2 + 2$ for $-3 \leq x \leq 3$



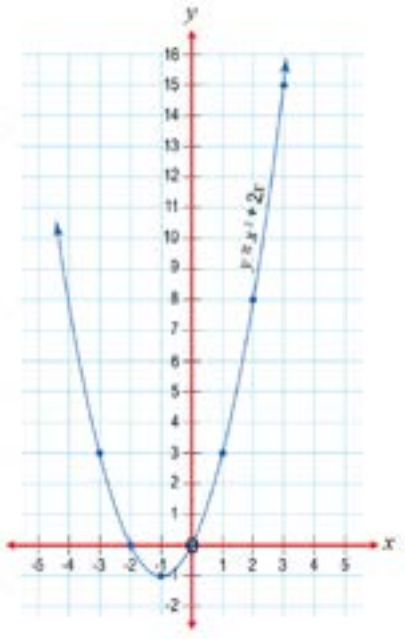
Question 2 $y = x^2 - 4$ for $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	5	0	-3	-4	-3	0	5



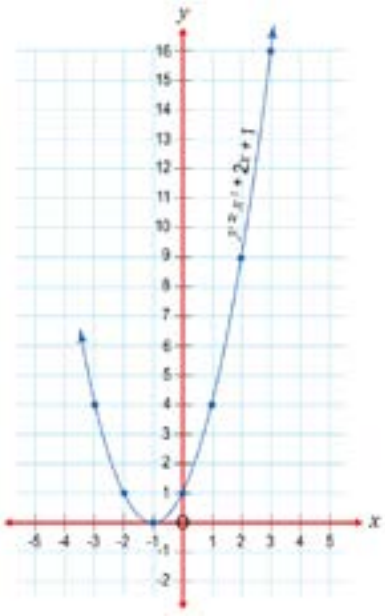
Question 3 $y = x^2 + 2x$ for $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	3	0	-1	0	3	8	15



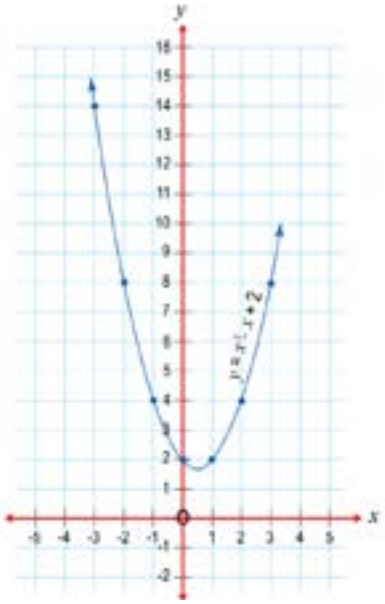
Question 4 $y = x^2 + 2x + 1$ for $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	4	1	0	1	4	9	16



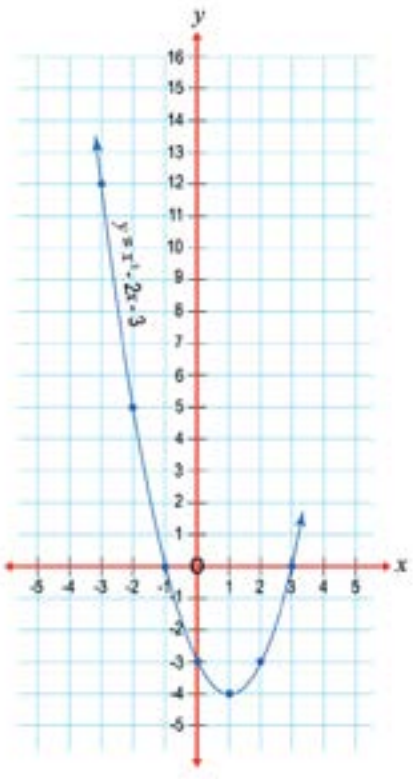
Question 5 $y = x^2 - x + 2$ for $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	14	8	4	2	2	4	8



Question 6 $y = x^2 - 2x - 3$ for $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	12	5	0	-3	-4	-3	0



6. **Question 1**

a) No x-intercepts

b) The y-intercept is at (0, 2)

Question 2

a) The x-intercepts are at (–2, 0) and (2, 0)

b) The y-intercept is at (0, –4)

Question 3

a) The x-intercepts are at (–2, 0) and (0, 0)

b. The y-intercept is at (0, 0)

Question 4

a) The x-intercept is at (–1, 0)

b) The y-intercept is at (0, 1)

Question 5

a) No x-intercepts

b) The y-intercept is at (0, 2)

Question 6

a) The x-intercepts are at (–1, 0) and (3, 0)

b) The y-intercept is at (0, –3)

Lesson 6

Topic: Patterns and algebra

Investigating x- and y- intercepts of a parabola

Lesson concepts

Representations — Non-linear models

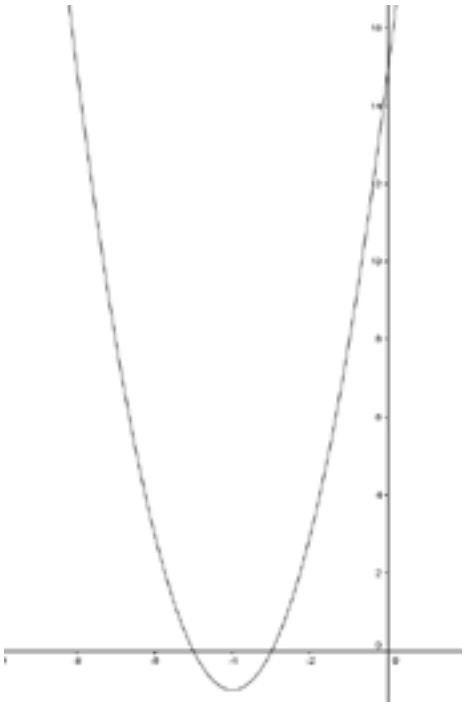
Lesson notes

Today students will:

- sketch non-linear functions using digital technology
- determine x- and y- intercepts of parabolic functions graphically.

Lesson answers

1. a) The x-intercepts are at (3, 0) and (4, 0).
   b) The y-intercept is at (0, 12).
2. Plot of $y = (x + 5)(x + 3)$



3. The x-intercepts are at (–5, 0) and (–3, 0). The y-intercept is at (0, 15).
4. In the function $y = (x + 5)(x + 3)$ the constants of 5 and 3 are opposite integer values to the positions of the x-intercepts (–5, 0) and (–3, 0).

   The expanded form of $y = (x + 5)(x + 3)$ is $y = x^2 + 5x + 15$. Notice that the constant 15 in the expanded function corresponds to the position of the y-intercept (0, 15).
5. a) $y = (x + 2)(x + 6)$

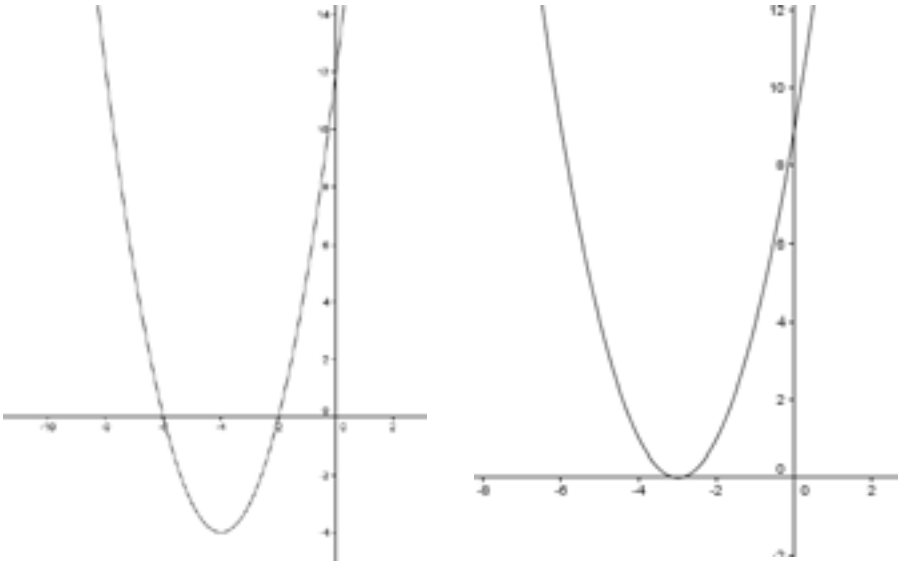
   As the constants in the function above are 2 and 6, the x-values for the x-intercepts will be –2 and –6. Therefore the x-intercepts are (–6, 0) and (–2, 0).

   The expanded function is $y = x^2 + 8x + 12$, therefore the y-value of the y-intercept will be at 12 and the y-intercept will be at (0, 12).

   b) $y = (x + 3)(x + 3)$

   As the constants in the function above are both 3, the x-value for the x-intercept will be –3. Therefore the x-intercept is (–3, 0).

   The expanded function is $y = x^2 + 6x + 9$, therefore the y-value of the y-intercept will be at 9 and the y-intercept will be at (0, 9).
6. Plots of $y = (x + 2)(x + 6)$ and $y = (x + 3)(x + 3)$ respectively:



7. **Sheet 4 — Sketching non-linear relations (attached).**

Expanding binomials

1. Expand the following binomials using the area model.

a) $(x + 2)(x + 4)$

	x	2
x	$x \times x = x^2$	$2 \times x = 2x$
4	$4 \times x = 4x$	$2 \times 4 = 8$

$= x^2 + 4x + 2x + 8$

$= x^2 + 6x + 8$

b) $(k + 8)(k + 3)$

	k	8
k	$k \times k = k^2$	$8 \times k = 8k$
3	$3 \times k = 3k$	$8 \times 3 = 24$

$= k^2 + 8k + 3k + 24$

$= k^2 + 11k + 24$

2. Expand the following binomials.

a) $(x - 4)(x + 2)$

$= x \times x + 2 \times x - 4 \times x - 4 \times 2$

$= x^2 + 2x - 4x - 8$

$= x^2 - 2x - 8$

b) $(g - 2)(g - 6)$

$= g \times g - 6 \times g - 2 \times g - 2 \times -6$

$= g^2 - 6g - 2g + 12$

$= g^2 - 8g + 12$

c) $(2x + 3)(x + 7)$

$= 2x \times x + 2x \times 7 + 3 \times x + 3 \times 7$

$= 2x^2 + 14x + 3x + 21$

$= 2x^2 + 17x + 21$

d) $(3r + 2)(6r - 6)$

$= 3r \times 6r + 3r \times -6 + 2 \times 6r + 2 \times -6$

$= 18r^2 - 18r + 12r - 12$

$= 18r^2 - 6r - 12$

3. Three students tried to expand the following binomial:

$$(x - 5)(x + 4)$$

They each came up with a different answer.

Answer 1: $2x - 20$	Answer 2: $x^2 - 9x - 20$	Answer 3: $x^2 - x - 20$

a) Which answer is correct?

$(x - 5)(x + 4)$

$= x \times x + 4 \times x - 5 \times x - 5 \times 4$

$= x^2 + 4x - 5x - 20$

$= x^2 - x - 20$

Answer 3 was the correct response.

b) Explain what type of errors were made by the other two students.

For Answer 1, I believe the student has simply added the x terms to give $2x$ and then multiplied the -5 and 4 to give 20, hence $2x - 20$.

For Answer 2, I believe the student has added $-5x$ to $-4x$ to give $9x$ when they should have added $-5x$ to $+4x$ to result in $-x$ as per Answer 3.

Sketching non-linear relations

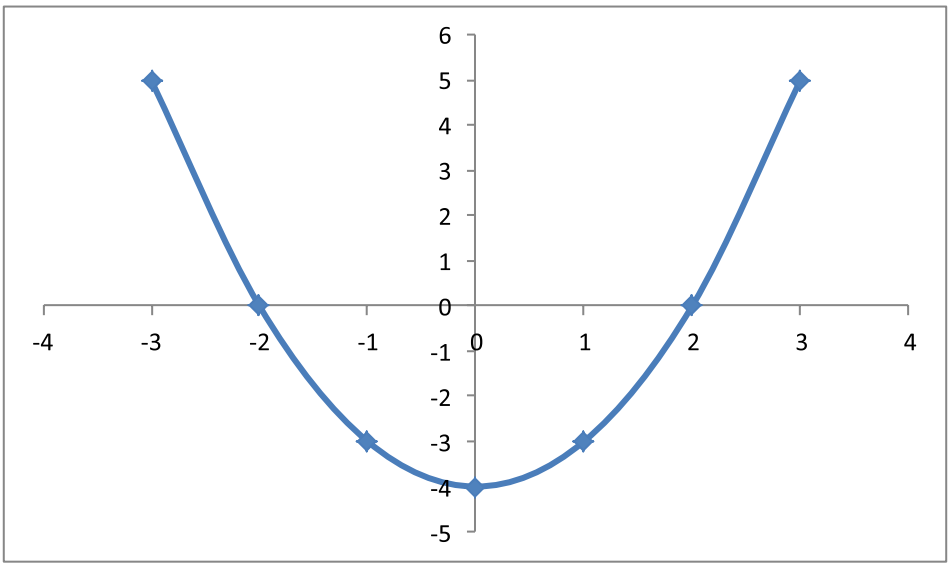
1. For the following function:

$$y = (x + 2)(x - 2)$$

a) Complete a table of values.

x	-3	-2	-1	0	1	2	3
y	5	0	-3	-4	-3	0	5

b) Sketch the plot on the grid below.



c) Identify and state the coordinates of the intercepts:

x-intercept/s: (–2, 0) and (2, 0)

y-intercept: (0, –4)